Modeling Interdependent Demand and Supply Uncertainties: Implications for Sourcing Decisions

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Motivation for the presentation

What is the impact of demand-supply interdependency when sourcing critical components from an unreliable supplier under uncertain demand?
Outline

Introduction
Newsvendor example
Capacity reservation option example
Conclusions
Introduction

• In the electronics industry, 60-70% of the cost of goods sold is attributed to the cost of purchased goods and services (Simchi-Levi, 2010)

• With intensified competition and increased demand uncertainty, sourcing has become a strategic process, supported with increasingly many decision models

• We study how sourcing decisions are affected by interdependent demand and supply, e.g.:
  – During an economic boom, demand for consumer electronics grows
  – On the supply side, semi-conductor capacity is limited
  ⇒ High demand may contribute to low supply capability, indicating a negative dependency between demand and supply
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Background of the newsvendor problem

- **The newsvendor** needs to decide how many newspapers to order for tomorrow, when both over and under ordering have a cost.
- In the original problem uncertainty pertains to demand only, but extensions have been proposed (cf. Khouja 1999, Qin et al. 2011).
- We consider a newsvendor facing uncertain demand and supply.

How many will I sell tomorrow?
What if I don’t get all the copies I ordered?
Formulation of the problem 1/2
Uncertainties

- The uncertain demand $D$ is characterized by probability distribution function $f_D$ and expected value $\mu_D$
- The newsvendor decides the order quantity $q$, of which a random proportion $Z \cdot q$ will be delivered
- $Z$ is the *stochastic yield*, described with $f_Z$ and $\mu_Z$
Formulation of the problem 2/2
The optimal order quantity

- The expected total cost $C_t$ consists of three separate elements:
  1. Holding costs (received order exceeds demand): $c_h \cdot \mathbb{E}[(Z \cdot q - D)^+]$
  2. Shortage costs (demand exceeds order): $c_s \cdot \mathbb{E}[(D - Z \cdot q)^+]$
  3. Ordering costs: $c_o \cdot \mu Z \cdot q$

- The optimal order quantity $q^*$ can be found with

  \[
  \frac{\partial C_t}{\partial q} = 0 \implies \frac{\partial}{\partial q} \mathbb{E}[|Z \cdot q - D|] = \frac{c_s - c_h - 2 \cdot c_o}{c_h + c_s} \cdot \mu Z
  \]

- …which eventually gives

  \[
  \int_{0}^{\infty} \int_{zq^*}^{D^*} f(z, d) \, dd \, dz = \frac{c_h + c_o}{c_h + c_s} \cdot \mu Z = \bar{\xi}
  \]

  Constant, if cost parameters and expected supply yield known

  Upper limits for $Z$ and $D$ (technical limitation, could be $\infty$)
  Optimal order
  Joint distribution function of $D$ and $Z$
Illustration of demand-supply interdependency

Bivariate normal joint distribution

- Assume that \( Z \) and \( D \) follow a bivariate normal distribution with \( D \sim N(100,50), Z \sim N(0.5,0.1) \)
Correlation coefficient $\rho$ clearly impacts the optimal order

- Integration of $\int_{z}^{z^+} \int_{d}^{D^+} f(z, d) \, dd \, dz = \frac{c_h + c_o}{c_h + c_s} \cdot \mu_Z = \xi$ numerically yields:

(a) $^\rho$ Optimal order quantities

(b) $^\rho$ Optimal expected costs
Key takeways of the newsvendor example

- My optimal order quantity varies significantly depending on the dependency strength and direction.
- If I have a high shortage cost, my business suffers a lot from negative correlation.
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Need for more complex setups

- Most companies are not simple newsvendors; they use different supply contracts, have multiple suppliers, etc.
- Also, many environments cannot be described with independent, or even linearly dependent demand and supply uncertainties
- Next example shows how:
  - Dependencies can be modeled using scenarios
  - A capacity reservation option can be used to hedge demand and supply risks
  - The results can be used to evaluate sourcing costs and risks
Scenarios can capture non-linear dependencies

- We use a Gumbel copula (cf. Embrechts et al. 2005) to create tail-dependent demand-supply scenarios.
Scenarios can utilize expert knowledge for both marginal and joint distributions

- Marginals are generated from percentile estimates for demand and supply (Hoyland & Wallace 2001):

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>0%</th>
<th>5%</th>
<th>50%</th>
<th>95%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply</td>
<td>0.60</td>
<td>0.75</td>
<td>0.90</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Demand</td>
<td>100k</td>
<td>150k</td>
<td>300k</td>
<td>500k</td>
<td>700k</td>
</tr>
</tbody>
</table>

- Dependency strength is a single-point estimate that corresponds to correlation coefficient
Unreliable and cheap fixed order vs. reliable and costly reservation option

- Consider two suppliers:
  - Unreliable (as in the newsvendor example), with order cost 4.5
  - Reliable, offering capacity option with reservation cost 1.0 and execution 5.0

Figure 3: The schematic description of the decision process.
MILP formulation of the problem

\[
\min_{q,r,e} C_0(r,q) + C_1(e,q) \quad (5.1)
\]

subject to

\[
C_0 = c_r r + c_o q \quad (5.2)
\]

\[
C_1 = \sum_n p_n \left[ c_h I_n^+ + c_s I_n^- + c_e e_n - (1 - Z_n) c_o q \right] \quad (5.3)
\]

\[
e_n \leq r, \forall n, \quad (5.4)
\]

\[
I_n = Z_n q + e_n - D_n, \forall n \quad (5.5)
\]

\[
I_n = I_n^+ - I_n^- \quad (5.6)
\]

\[
r, q, e_i, n \in \mathbb{N} \quad (5.7)
\]
Example results from running the model with 2 500 scenarios

Scenario parameters:
E[D]=300 000
E[Z]=90%

Optimization results:
q* = 179 200
r* = 229 700
⇒ Total capacity 408 900

Histogram of costs in each scenario with optimal strategy

- Conditional-Value-at-Risk (5%)
  3 500 000
- Expected costs
  1 730 000
The model is solved for *same marginals* under *different dependency structures*.

Dependency strength grows from *Linear* to *Tail-dependent*.
Costs increase with dependency strength, risks increase especially in tail-dependency.

Optimal strategy when $\rho=0\rightarrow0.5$:
- Fixed order from 200k $\rightarrow$ 180k
- Reservation from 210k $\rightarrow$ 235k

**Figure 5:** Optimization results when demand-supply dependency varies: expected costs (left) and CVaR$_{5\%}$ values (right).
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Interdependencies matter!

- No dependency vs. strong dependency impacts the optimal order and reservation quantities in the scale of tens of percents.
- Risk level (CVaR-5%) also grows tens of percents, expected costs a lot less.
- Further analyses showed that:
  - Capacity reservation option can cut expected costs over 10% compared to fixed orders only.
  - Common components hedge demand risks effectively only when end product demands are not positively dependent.
  - Differences in dependency structure (linear vs. tail dependent) can increase worst-case risks significantly (bankruptcy, delay of flagship product launch).
References


